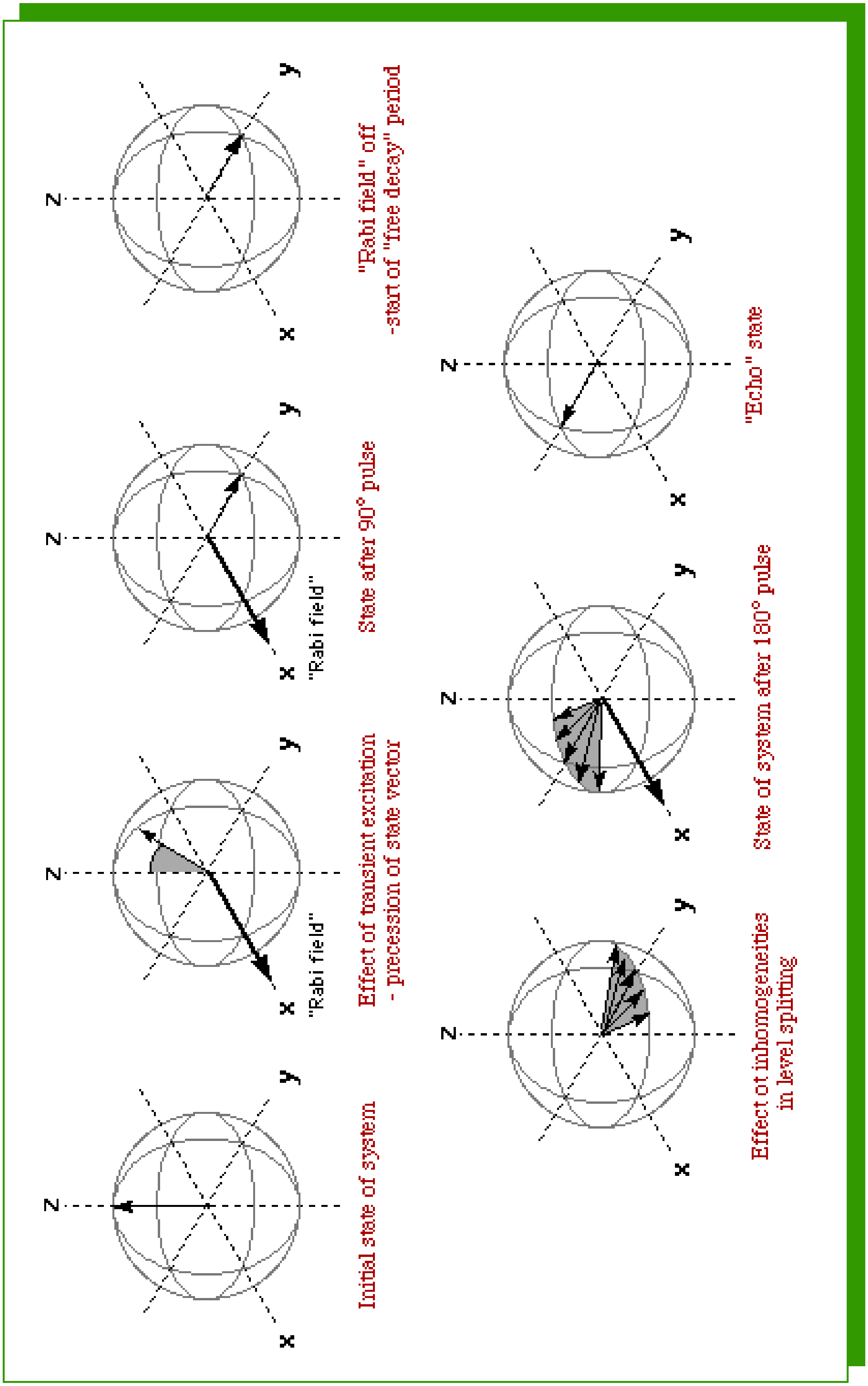
APPLIED PHYSICS 216
OPTICAL PHYSICS AND QUANTUM ELECTRONICS
ASSIGNMENT 2
SPRING TERM 1999-2000

Due Friday, April 14, 2000.

1. The single-mode self-consistent laser equation $\dot{I}_n \approx 2I_n \left[\alpha_n - \beta_n I_n\right]$ (*i.e.*, Equation [VI-25b] in *Semiclassical Laser Theory*) yield **two** possible steady state solutions. Following the discussion in Equations [VI-43] through [VI-48] **find** and **discuss** the stability criterion for each of the stationary solutions.

2. In many instances, the matter interacting with radiation may be characterized as **inhomogeneous**, that is, the various atoms in the medium have level seperations which are statistically distributed in some manner (a canonical example is the line broadening of the Cr^{3+}. spectra in ruby where the level splittings due to the local crystal field are modulated by an inhomogeneous strain distribution). The hallmark of optical inhomogeneity is that the width of spectral absorption peaks (or equivalently, laser gain curves) due to the distribution of level splittings is significantly greater than the width associated with all relaxation mechanisms. At a deeper level, inhomogeneous broadening, in contrast to relaxation broadening, is a deterministic or dynamic process which may be reversible under certain circumstances. The "phonon echoes" discussed in *Review of Basic Quantum Mechanics: Two-Level Quantum Systems:* is one important means of demonstrating reversibility. In the figure below, we attempted a depiction of the time evolution of the states of a collective system of two-level atoms which have a static distribution of splittings. The essential equivalence of Equations [III-17] and [III-20] is the basis of this depiction.

   The first frame depicts the system in its initial state where, essentially, all of the atoms are in their lowest energy state. The second frame shows a representation of the collective state in the **rotating frame** after an optical excitation has been turned on. Of course, the excitation ("Rabi field") is fixed in the rotating frame and the atomic states tend to precess around it. After a time $\tau_{\pi/2}$ the optical excitation is turn off and the state of the system is depicted in frames 3 and 4. We suppose that system is inhomogeneous in the sense that it is characterized by some distribution of level splittings $E_a - E_b$ -- *i.e.*, each two-level atom has a slight different level splitting which differs from some mean value. Frame 5 depicts the state of system after some "free decay" time τ_{decay} and illustrates how the individual state vectors ***fan out*** from the average. At that point a second optical pulse of length τ_{π} is applied and causes the individual state vectors to precess into the configuration depicted in frame 6 (middle right). Frame 7 depicts the situation after an additional time interval τ_{decay} in which the individual state vectors ***fan together*** to give an optical ***echo***.


Initial state of system
Effect of transient excitation
- precession of state vector
"Rabi field"
State after 90° pulse
"Rabi field"
"Rabi field" off
-start of "free decay" period
Effect of inhomogeneities
in level splitting
State of system after 180° pulse
"Echo" state
x
y
z

After this long introduction, your tasks are:

a. Find the requisite **energy** of the $\pi/2$ (90˚) and π (180˚) pulses.

b. In order for things to work as advertised above, what restrictions apply? What can you say about restrictions on $\tau_{\pi/2}$, τ_{π}, τ_{decay}, or any other critical parameter?

c. Clearly explain and discuss what might be **experimentally** observed over the course of events depicted.

d. Discuss what would be observed if there were to a repeated sequence of frames 4 through 7 - *i.e.*. 180˚ pulses interspersed with free evolution periods.

3. Since $\left|U_n(z)\right|^2 = 1$ for a runninng wave, Equation [VI-21 in *Semiclassical Laser Theory* simplifies to

$$\mathcal{P}_n\left(\omega_{ab},\omega_n,\gamma_{ab}\right) = -i\ {}^2\hbar^{-1}\mathcal{E}_n\ \bar{N}\ \frac{\mathcal{D}\left(\omega_{ab}-\omega_n;\gamma_{ab}\right)}{1+\bar{I}_n\ \mathcal{L}\left(\omega_{ab}-\omega_n;\gamma_{ab}\right)}$$

where $\bar{N}$ is the average population inversion density and

$$\bar{I}_n = \frac{{}^2\ \mathcal{E}_n^2}{\hbar^2}\frac{1}{2}\left(\frac{1}{\gamma_a}+\frac{1}{\gamma_b}\right)\frac{1}{\gamma_{ab}} = I_n\frac{\left(\gamma_a+\gamma_b\right)}{\gamma_{ab}}$$

is one way to write the dimensionless intensity of the nth cavity mode. For an inhomogeneous medium characterized by a spread in line centers, the mode polarization may be expressed as

$$\mathcal{P}_n\left(\omega_n,\gamma_{ab}\right) = \int d\omega_{ab}\ P\left(\omega_{ab}\right)\mathcal{P}_n\left(\omega_{ab},\omega_n,\gamma_{ab}\right)$$

where $P\left(\omega_{ab}\right)$ is theprobability distribution associated with the inhomogeneity. In many instances, the inhomogeniety may be described as Gaussian -- *i.e.* where

$$P\left(\omega_{ab}\right) = \frac{1}{\Delta\omega_{ab}\sqrt{\pi}}\exp\left[-\left(\frac{\omega_{ab}-\left\langle\omega_{ab}\right\rangle}{\Delta\omega_{ab}}\right)^2\right]$$

a. In general, it is a difficult analytic task to convolve the line broadening mechanisms to obtain a frequency dependent absorption or small-signal gain curve in all but the two extreme cases of $\Delta\omega_{ab} >> \gamma_{ab}$ and $\Delta\omega_{ab} << \gamma_{ab}$. To see how broadening mechanism affect line shape, I would like you to use *MATLAB* or *Mathematica* to build a numerical integration routine for plotting small-signal gain as function of frequency for an arbitrary value of the ratio $\Delta\omega_{ab}/\gamma_{ab}$. To test your routine, see if it produces the expected results in the extreme cases. To get something useful, plot out two or three line shapes for intermediate values of $\Delta\omega_{ab}/\gamma_{ab}$.[1]

b. In one form of **saturation spectroscopy**, a saturating or pump signal at a frequency ω_{pump} produces a change in the population difference between atomic levels which may be measured by observing the induced small-signal polarization at a probe frequency ω_{probe} -- *i.e.*

$$\mathcal{P}_{\text{probe}}\left(\omega_{\text{probe}}\right) = -i \quad {}^{2}\,\hbar^{-1}\mathcal{E}_{\text{probe}}\ \overline{N}\int d\omega_{ab}\ P\left(\omega_{ab}\right)\frac{\mathcal{D}\left(\omega_{ab}-\omega_{\text{probe}};\gamma_{ab}\right)}{1+\bar{I}_{\text{pump}}\ \mathcal{L}\left(\omega_{ab}-\omega_{\text{pump}};\gamma_{ab}\right)}$$

So called ***spectral hole burning*** is one of the more interesting features of this simple form of saturation spectroscopy. Hole burning occurs when there is significant saturation -- *e.g.* when $\bar{I}_{\text{pump}} \geq 1$ -- and is manifested by a notch or dip in the peak of the probe absorption peak. I would like you to extend the numerical integration routine that you developed above to give the imaginary part of the probe susceptibility as a function of ω_{probe}, ω_{pump}, $\bar{I}_{\text{pump}}$, and $\Delta\omega_{a,b}/\gamma_{a,b}$. Experiment with your routine to establish the conditions under which hole burning is observable in a degenerate pump-probe -- *i.e.* $\omega_{\text{probe}} = \omega_{\text{pump}}$ -- experiment. Report on your observations and plot at least one probe susceptibility as a function of ω_{probe} which shows hole burning.[2]

4. Show that the dielectric susceptibility embedded in Equation [III-19] of *Review of Basic Quantum Mechanics: Two-Level Quantum Systems,* satisfies the Kramers-Kronig relations [2] in the **limit of negligible saturation**. At higher power levels the Kramers-Kronig relations are no longer strictly valid. Do you have any idea why they fail?

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1 Please hand in a hard copy of your routine as well as copies of your line shape plots.

2 The Kramers-Kronig relations:

$$\chi''\left(\omega\right) = -\frac{1}{\pi}\,\mathcal{P}\int_{-\infty}^{\infty}\frac{\chi'\left(\omega'\right)}{\omega'-\omega}\,d\omega'$$

$$\chi'\left(\omega\right) = \frac{1}{\pi}\,\mathcal{P}\int_{-\infty}^{\infty}\frac{\chi''\left(\omega'\right)}{\omega'-\omega}\,d\omega'$$

where

5. The subject is **transients in gain-switched lasers.** Consider the following set of model equations for laser operation:

$$\frac{dn}{dt} = -\frac{n}{\tau_p} + \left[\mathbf{N}/\mathbf{N}_{th}\right]\frac{n}{\tau_p}$$

$$\frac{d\mathbf{N}}{dt} = \frac{\mathbf{N}_0}{\tau_{sp}} - \frac{\mathbf{N}}{\tau_{sp}} - 2\left[\mathbf{N}/\mathbf{N}_{th}\right]\frac{n}{\tau_p}$$

where:
- $n \equiv$ the photon-number density
- $\mathbf{N} \equiv$ the population inversion (difference)
- $\mathbf{N}_{th} \equiv$ the threshold population inversion (difference)
- $\mathbf{N}_0 \equiv 2R\,\tau_{sp} - \left(N_1 + N_2\right) =$ "small signal" inversion
- $R \equiv$ "pumping" rate
- $\tau_p \equiv$ the photon "lifetime" which accounts for photon leakage from the resonator
- $\tau_{sp} \equiv$ the inverse spontaneous emission rate

These equations can be written in the dimensionless form

$$\frac{dX}{ds} = -X + X\,Y$$

$$\frac{dY}{ds} = a\left(Y_0 - Y\right) - 2X\,Y$$

Write a computer program to solve these two equations for both switching on and switching off the laser. Assume that Y_0 is switched from 0 to 2 to turn the laser on, and from 2 to 0 to turn it off. Assume further that an initially very small photon flux corresponding to $X = 10^{-5}$ starts the oscillation at $t = 0$. Speculate on the possible origin of this flux. Determine the switching transient times for $a = 10^{-3}$, 1, and $a = 10^{3}$. Comment on the validity of the model equations and the significance of your results.

6. Consider a laser characterized by running-wave (unidirectional) mode functions of the form $U_n\left(z\right) = \exp\left(i\,k_n\,z\right)$

   a. Evaluate the third-order approximation for density matrix -- see Equations [VII-34] through [VII-37].

$$\mathcal{P}\int_{-\infty}^{+\infty} A\left(x\right)\,dx = \lim_{\delta\to 0}\left\{\int_{-\infty}^{-\delta} A\left(x\right)\,dx + \int_{+\delta}^{+\infty} A\left(x\right)\,dx\right\}$$

b. Calculate the linear and saturation coefficients corresponding to those calculated for standing-wave modes in the *Semiclassical Laser Theory* lectures.