Multistage Amplifier Frequency Response

* Summary of frequency response of single-stages:

  CE/CS: suffers from Miller effect

  CC/CD: “wideband” -- see Section 10.5

  CB/CG: “wideband” -- see Section 10.6

  (wideband means that the stage operates to near the frequency limit of the device ... f_T)

* How to find the Bode plot for a general multistage amplifier?

  can’t handle *n* poles and *m* zeroes analytically --> SPICE!

  develop analytical tool for an important special case:

  *** no zeroes**

  *** exactly one “dominant” pole** ($\omega_1 << \omega_2, \omega_3, \ldots, \omega_n$)

$$\frac{V_{out}}{V_{in}} = \frac{A_o}{(1 + j(\omega/\omega_1))(1 + j(\omega/\omega_2))(\ldots)(1 + j(\omega/\omega_n))}$$

  (the example shows a voltage gain ... it could be I_{out}/V_{in} or V_{out}/I_{in})

Finding the Dominant Pole

* Multiplying out the denominator:

$$\frac{V_{out}}{V_{in}} = \frac{A_o}{1 + b_1 j\omega + b_2 (j\omega)^2 + \ldots + b_n (j\omega)^n}$$

The coefficient b_1 originates from the sum of $j\omega/\omega_i$ factors --

$$b_1 = \frac{1}{\omega_1} + \frac{1}{\omega_2} + \ldots + \frac{1}{\omega_n} = \sum_i^n \frac{1}{\omega_i} \approx \frac{1}{\omega_1}$$

Therefore, if we can estimate the linear coefficient b_1 in the demoninator polynomial, we can estimate of the dominant pole

Procedure: see P. R. Gray and R. G. Meyer, *Analysis and Design of Analog Integrated Circuits*, 3rd ed., Wiley, 1994, pp. 502-504.

1. Find circuit equations with current sources driving each capacitor

2. Denominator polynomial is determinant of the matrix of coefficients

3. b_1 term comes from a sum of terms, each of which has the form:

$$R_{Tj}\, C_j$$

where C_j is the j^{th} capacitor and R_{Tj} is the Thévenin resistance across the j^{th} capacitor terminals (with all capacitors open-circuited)

Open-Circuit Time Constants

* The dominant pole of the system can be estimated by:

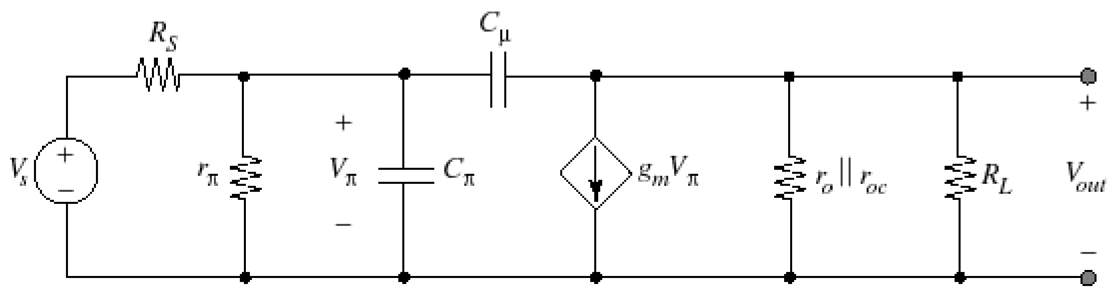
$$\omega_1 \approx \frac{1}{b_1} = \left(\sum_i^n R_{Tj} C_j \right)^{-1} = \left(\sum_1^n \tau_j \right)^{-1},$$

where $\tau_j = R_{Tj} C_j$ is the **open-circuit time constant** for capacitor C_j

* This technique is valuable because it estimates the contribution of each capacitor to the dominant pole frequency *separately* ... which enables the designer to understand what part of a complicated circuit is responsible for limiting the bandwidth of the amplifier.

Example: Revisit CE Amplifier

* Small-signal model:



* Apply procedure to each capacitor separately

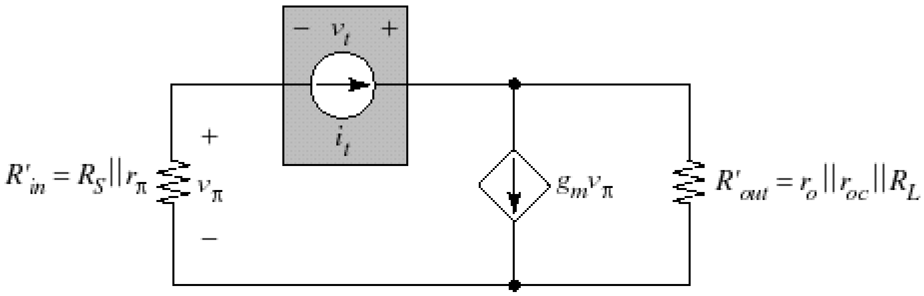
1. C_π's Thévenin resistance is found by inspection as the resistance across its terminals with all capacitors open-circuited:

$$R_{T\pi} = R_S \| r_\pi = R_{in}' \text{ --> } \tau_{C_{\pi o}} = R_{T\pi} C_\pi$$

2. C_μ's Thévenin resistance is *not* obvious --> must use test source and network analysis

Time Constant for C_μ

* Circuit for finding $R_{T\mu}$



v_π is given by:

$$v_\pi = -i_t(R_s || r_\pi) = -i_t R_{in}'$$

v_o is given by:

$$v_o = -i_o R_{out}' = (i_t - g_m v_\pi) R_{out}' = i_t(g_m R_{in}' + 1) R_{out}'$$

v_t is given by:

$$v_t = v_o - v_\pi = i_t((1 + g_m R_{in}') R_{out}' + R_{in}')$$

solving for $R_{T\mu} = v_t / i_t$

$$R_{T\mu} = R_{in}' + R_{out}' + g_m R_{in}' R_{out}'$$

$$\tau_{C_{\mu o}} = R_{T\mu} C_\mu = (R_{in}' + R_{out}' + g_m R_{in}' R_{out}') C_\mu$$

Estimate of Dominant Pole for CE Amplifier

* Estimate dominant pole as inverse of sum of open-circuit time constants

$$\omega_1^{-1} = (R_{T\pi}C_\pi + R_{T\mu}C_\mu) = R_{in}'C_\pi + (R_{in}' + R_{out}' + g_m R_{in}' R_{out}')C_\mu$$

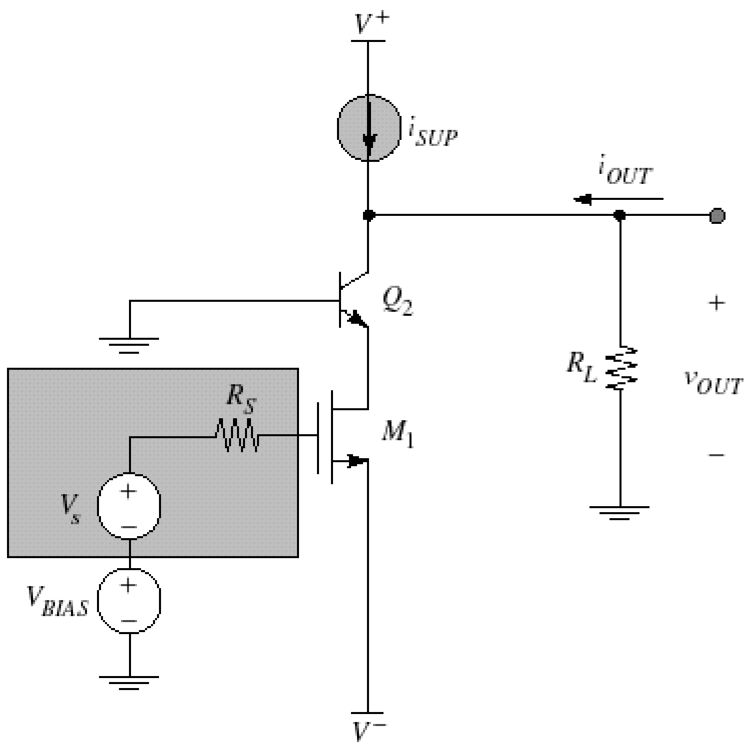
inspection --> identical to "exact" analysis (which also assumed $\omega_1 \ll \omega_2$)

* Advantage of open-circuit time constants: *general* technique

Example: include C_{cs} and estimate its effect on ω_1

Multistage Amplifier Frequency Response

* Applying the open-circuit time constant technique to find the dominant pole frequency -- use CS/CB cascode as an example



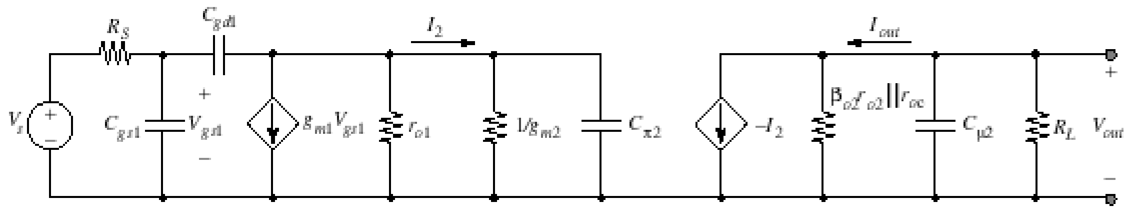
* Systematic approach:

  1. two-port small-signal models for each stage (*not* the device models!)

  2. carefully add capacitances across the appropriate nodes of two-port models, which may not correspond to the familiar device configuation for some models

Two-Port Model for Cascode

* The base-collector capacitor $C_{\mu 2}$ is located between the output of the CB stage (the collector of Q_2) and small-signal ground (the base of Q_2)



We have omitted C_{db1}, which would be in parallel with $C_{\pi 2}$ at the output of the CS stage, and C_{cs2} which would be in parallel with $C_{\mu 2}$. In addition, the current supply transistor will contribute additional capacitance to the output node.

* Time constants

$$\tau_{C_{gs1o}} = R_S C_{gs1}$$

$$\tau_{C_{gd1o}} = (R_{in}' + R_{out}' + g_{m1}R_{in}'R_{out}')C_{gd1}$$

where $R_{in}' = R_S$ and $R_{out}' = r_{o1}||\left(\frac{1}{g_{m2}}\right) \approx \frac{1}{g_{m2}}$

Since the output resistance is only $1/g_{m2}$, the Thévenin resistance for C_{gd1} is not magnified (i.e., the Miller effect is minimal):

$$\tau_{C_{gd1o}} = \left(R_S + \frac{1}{g_{m2}} + \left(\frac{g_{m1}}{g_{m2}}\right)R_S\right)C_{gd1} \approx R_S(1 + g_{m1}/g_{m2})C_{gd1}$$

Cascode Frequency Response (cont.)

* The base-emitter capacitor of Q_2 has a time constant of

$$\tau_{C_{\pi 2o}} = \left(\frac{1}{g_{m2}}\right) C_{\pi 2}$$

* The base-collector capacitor of Q_2 has a time constant of

$$\tau_{C_{\mu 2o}} = (\beta_{o2} r_{o2} \| r_{oc} \| R_L) C_{\mu 2} \approx R_L C_{\mu 2}$$

* Applying the theorem, the dominant pole of the cascode is approximately

$$\omega_{3db}^{-1} \approx \tau_{C_{gs1o}} + \tau_{C_{gd1o}} + \tau_{C_{\pi 2o}} + \tau_{C_{\mu 2o}}$$

$$\omega_{3db}^{-1} \approx R_S C_{gs1} + R_S (1 + g_{m1} / g_{m2}) C_{gd1} + \left(\frac{1}{g_{m2}}\right) C_{\pi 2} + R_L C_{\mu 2}$$

Gain-Bandwidth Product

* A useful metric of an amplifier's frequency response is the product of the low-frequency gain $|A_{vo}|$ and the 3 dB frequency ω_{3dB}

  For the cascode, the gain is $|A_{vo}| = |-g_{m1}R_L|$ and the gain-bandwidth product is

$$|A_{vo}|\omega_{3dB} \approx \frac{g_{m1}R_L}{R_S C_{gs1} + R_S(1 + g_{m1}/g_{m2})C_{gd1} + \left(\frac{1}{g_{m2}}\right)C_{\pi 2} + R_L C_{\mu 2}}$$

* If the voltage source resistance is small, then

$$|A_{vo}|\omega_{3dB} \approx \frac{g_{m1}R_L}{(C_{\pi 2}/g_{m2} + R_L C_{\mu 2})}$$

  which has the same form as the common-base gain-bandwidth product (and which is *much* greater than the Miller-degraded common-source)